THERMOSTRESSED STATE AT A LOCAL THERMAL CONTACT IN TURNING FRICTION

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Processes of friction heating at a local thermal contact in turning friction are studied.

Turning friction is usually understood as a relative rotation of contacting bodies around an axis coinciding with the common normal to the contact surface. The temperature at a steady-state thermal contact was studied in [1]. In the present work the authors consider the development of the process of friction heating starting at a moment which is assumed to be the initial time.

1. Let two semi-infinite elastic bodies related to a cylindrical coordinate system (r, θ, z) have a circular contact area $0 \le r \le a$ and rotate with relative angular velocity ω. In this case, friction forces obeying Amonton's law [2] give rise to heat sources distributed over the contact area. It is assumed that there is no heat transfer from the free surface of the contacting bodies and that all the heat generated in the contact region is absorbed by the contacting bodies. The intensity of the heat flux towards one of the bodies in the friction pair is

$$q(r) = \gamma f \omega r p(r), \quad r \le a. \tag{1}$$

Here γ is the heat flux separation coefficient [3]; $p(r)$ is the contact pressure in Hertz's problem [2]

$$p(r) = p_0 \sqrt{\left(1 - \left(\frac{r}{a}\right)^2\right)}, \tag{2}$$

where $p_0 = 3P/2\pi a^2$; P is the pressing force.

The temperature field of a semi-infinite body caused by heat flux (1) at an arbitrary time t is determined by solution of the parabolic heat conduction equation

$$\Delta T = \frac{1}{k}\frac{\partial T}{\partial t} \tag{3}$$

with the initial condition

$$T = 0 \quad \text{at} \quad t = 0, \tag{4}$$

boundary conditions

$$K\frac{\partial T}{\partial z} = \begin{cases} -q_0 r \sqrt{\left(1 - \left(\frac{r}{a}\right)^2\right)}, & r \le a, \ z = 0, \\ 0, & r > a, \ z = 0, \end{cases} \tag{5}$$

$$\frac{\partial T}{\partial r} = 0 \quad \text{at} \quad r = 0, \ z \ge 0 \tag{6}$$

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and the regularity condition

$$\left\{T, \frac{\partial T}{\partial r}\right\} \to 0 \quad \text{at} \quad r^2 + z^2 \to \infty . \tag{7}$$

In Eq. (5)

$$q_0 = \gamma f \omega p_0 . \tag{8}$$

Without mass forces and ignoring inertia effects, the components of the displacement vector u_i^t and the stress tensor σ_{ij}^t caused by the nonuniform temperature distribution $T(r, z, \tau)$ are related to the thermoelastic displacement potential Ψ^t and the Lava function L by the known relationship [4]. The functions Ψ^t and L are solutions of the differential equations

$$\Delta \Psi^t = \beta T , \tag{9}$$

$$\Delta \Delta L = 0 . \tag{10}$$

Here $\beta = \alpha(1 + \nu)/(1 - \nu)$.

In the case considered the components of the displacement vector and the stress tensor should satisfy the boundary conditions

$$u_r^t = 0 \quad \text{at} \quad r = 0 , \quad z \geq 0 , \tag{11}$$

$$\sigma_{rz}^t = \sigma_{zz}^t = 0 \quad \text{at} \quad z = 0 , \quad r \geq 0 \tag{12}$$

and the descent conditions

$$\left\{u_r^t , u_z^t\right\} \to 0 \quad \text{at} \quad r^2 + z^2 \to \infty . \tag{13}$$

2. A solution of the boundary-value problem determined by equations and relations (1)-(8) is obtained by successive use of the Hankel integral tranform of zero order with respect to the variable r and of the Laplace transform with respect to t. As a result

$$T(\rho , Z , \text{Fo}) = \Lambda \int_0^\infty \varphi(\xi) \Phi(\xi , Z , \text{Fo}) J_0(\xi\rho)\, d\xi . \tag{14}$$

$$\varphi(\xi) = \frac{2\pi}{\xi^2} \sum_{m=0}^{\infty}{}' (-1)^m (m + 1) [J_{m+1}(\xi/2)]^2 ,$$

$$\Phi(\xi , Z , \text{Fo}) = \frac{1}{2}\left[\exp(-\xi Z)\, \text{erfc}\left(\frac{Z}{2\sqrt{\text{Fo}}} - \xi\sqrt{\text{Fo}}\right) - \right.$$

$$\left. - \exp(\xi Z)\, \text{erfc}\left(\frac{Z}{2\sqrt{\text{Fo}}} + \xi\sqrt{\text{Fo}}\right)\right] , \quad \rho = \frac{r}{a} , \quad Z = \frac{z}{a} , \quad \text{Fo} = \frac{kt}{a^2} ,$$

$$\Lambda = \frac{q_0 a^2}{K} , \quad \text{erfc}(\cdot) = 1 - \text{erf}(\cdot) ;$$

the prime at $\sum{}'$ means that the term at $m = 0$ is taken with the factor 1/2.

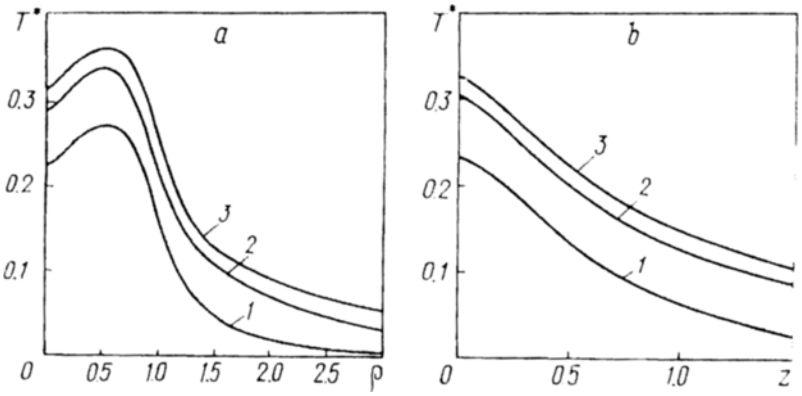

Fig. 1. Variation of dimensionless temperature T^* on surface z (a) and along axis of rotation $\rho = 0$ (b): 1) Fo = 1; 2) 10; 3) 100.

Under steady-state conditions of heat release at Fo $\to \infty$ the function Φ tends to $\exp(-\xi Z)$, and it follows from (14) that

$$T_\infty(\rho, Z) = \Lambda \int_0^\infty \varphi(\xi) \exp(-\xi Z) J_0(\xi\rho)\, d\xi .$$

The temperature at the center of the contact area, i.e., at $\rho = 0$ and $Z = 0$, is defined by the expression

$$T_0 = \Lambda \int_0^\infty \varphi(\xi)\, d\xi . \tag{15}$$

Using the value of the integral from [5]

$$\int_0^\infty \xi^{-2} J_{m+1}^2(\xi/2)\, d\xi = \frac{2}{\pi(2m+1)(2m+3)},$$

we transform relation (15) to the form

$$T_0 = 4\Lambda \sum_{m=0}^{\infty} \frac{(-1)^m (m+1)}{(2m+1)(2m+3)} = \frac{\Lambda}{3}. \tag{16}$$

It should be noted that formula (16) for the steady-state temperature at the center of the contact area was originally obtained in [1].

Plots of the relative temperature $T^* = T/\Lambda$ (14) on the surface $z = 0$ and along the axis of rotation $\rho = 0$ with fixed parameter Fo = 1; 10; 100 are presented in Fig. 1.

Unlike the case of mutual sliding of bodies, in the case of turning, the development of temperature fields has characteristic features. While in the case of mutual sliding the maximum surface temperature is at the center of the heating region [6], in the case of turning the maximum temperature develops inside the contact region at a distance from the axis of rotation equal to about a half of the contact radius. In particular, at Fo = 100 the maximum T^* of 0.36 is at the point $\rho = 0.51$. Outside the contact region ($\rho > 1$) the temperature declines rapidly.

3. Solutions of Eqs. (9) and (10) obtained with the help of Hankel and Laplace the integral transforms are written in the form

$$\Phi(\rho, Z, \mathrm{Fo}) = \Lambda\beta \int_0^\infty \varphi(\xi)\, \Phi_1(\xi, Z, \mathrm{Fo})\, J_0(\xi\rho)\, d\xi ,$$

$$L(\rho, Z, \mathrm{Fo}) = -Ca^3 \int_0^\infty \varphi(\xi)\, \Phi_2(\xi, 0, \mathrm{Fo}) \left(\frac{2\nu - 1}{\xi} + Z\right) \exp(-\xi Z)\, J_0(\xi\rho)\, d\xi . \tag{17}$$

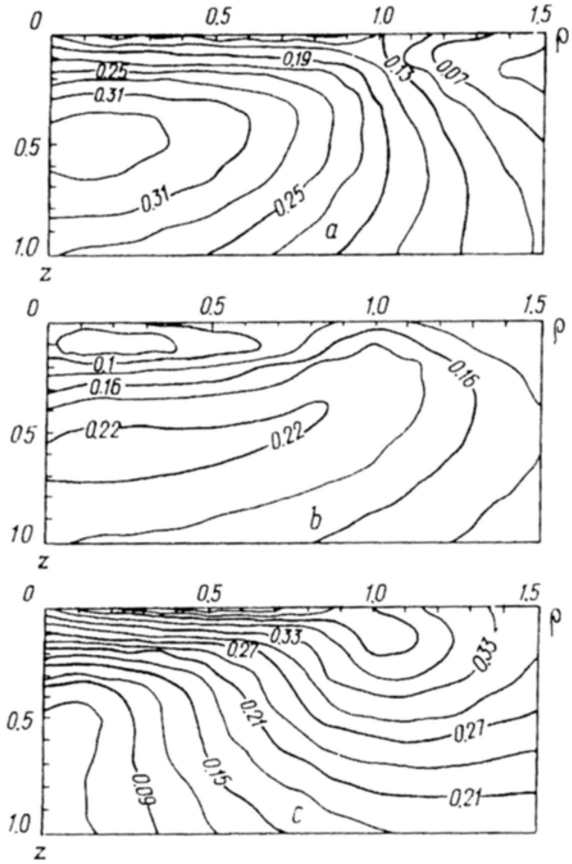

Fig. 2. Isotherms of dimensionless stress J_2^* at Fo = 10: a) $w_t = 0$; b) 3; c) 8.

Here

$$\Phi_1(\xi, Z, \mathrm{Fo}) = \exp(-\xi Z)\left(\frac{\mathrm{Fo}}{2} - \frac{Z}{4\xi} - \frac{1}{4\xi^2}\right)\operatorname{erfc}\left(\frac{Z}{2\sqrt{\mathrm{Fo}}} - \xi\sqrt{\mathrm{Fo}}\right) -$$

$$- \exp(\xi Z)\left(\frac{\mathrm{Fo}}{2} + \frac{Z}{4\xi} - \frac{1}{4\xi^2}\right)\operatorname{erfc}\left(\frac{Z}{2\sqrt{\mathrm{Fo}}} + \xi\sqrt{\mathrm{Fo}}\right) +$$

$$+ \xi^{-1}\sqrt{\left(\frac{\mathrm{Fo}}{\pi}\right)}\exp(-\xi^2\mathrm{Fo})\left[\exp\left(-\frac{\xi^2}{4\mathrm{Fo}}\right) - \exp(-\xi Z)\right] -$$

$$- \exp(-\xi Z)\left(\mathrm{Fo} - \frac{1}{2\xi^2}\right)\operatorname{erf}(\xi\sqrt{\mathrm{Fo}});$$

$$\Phi_2(\xi, Z, \mathrm{Fo}) = \left(\frac{Z}{4\xi} - \frac{\mathrm{Fo}}{2}\right)\left[\exp(-\xi Z)\operatorname{erfc}\left(\frac{Z}{2\sqrt{\mathrm{Fo}}} - \xi\sqrt{\mathrm{Fo}}\right) +\right.$$

$$\left.+ \exp(\xi Z)\operatorname{erfc}\left(\frac{Z}{2\sqrt{\mathrm{Fo}}} + \xi\sqrt{\mathrm{Fo}}\right)\right] + \exp(-\xi Z)\left(\mathrm{Fo} - \frac{1}{2\xi^2}\right)\operatorname{erf}(\xi\sqrt{\mathrm{Fo}}) +$$

$$+ \xi^{-1}\sqrt{\left(\frac{\mathrm{Fo}}{\pi}\right)}\exp[-(\xi^2\mathrm{Fo} + \xi Z)]; \quad C = p_0 w_t;$$

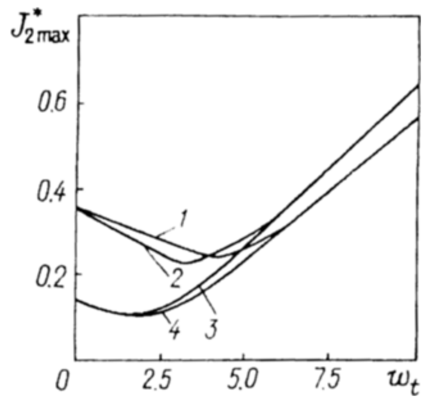

Fig. 3. Plot of $J^*_{2\max}$ versus the parameter w_t: 1, 3) Fo = 1; 2, 4) 10; 1, 2) $\rho \le 1.5$; $0 \le Z \le 1$; 3, 4) $\rho \le 1.5$, $Z = 0$.

$$w_t = \frac{2(1+\nu)\mu\alpha\gamma f\omega a^2}{(1-\nu)K}. \tag{18}$$

The temperature stress field induced in an elastic body by turning friction is obtained using relations (17). The unknown components of the temperature stress tensor are expressed in terms of improper integrals as follows:

$$\sigma^t_{rr} = C\int_0^\infty \varphi(\xi)\left\{\Phi_1(\xi, Z, \text{Fo})\left[\frac{\xi}{\rho}J_1(\xi\rho) - \xi^2 J_0(\xi\rho)\right] -\right.$$

$$\left.- \xi\exp(-\xi Z)\Phi_2(\xi, 0, \text{Fo})\left[(2\xi - \xi^2 Z)J_0(\xi\rho) + (2\nu - 2 + \xi Z)\frac{J_1(\xi\rho)}{\rho}\right]\right\}d\xi - CT^*,$$

$$\sigma^t_{\theta\theta} = -C\int_0^\infty \varphi(\xi)\left\{\Phi_1(\xi, Z, \text{Fo})\frac{\xi}{\rho}J_1(\xi\rho) +\right.$$

$$\left.+ \xi\exp(-\xi Z)\Phi_2(\xi, 0, \text{Fo})\left[2\nu\xi J_0(\xi\rho) - (2\nu - 2 + \xi Z)\frac{J_1(\xi\rho)}{\rho}\right]\right\}d\xi - CT^*, \tag{19}$$

$$\sigma^t_{zz} = C\int_0^\infty \varphi(\xi)\left[\Phi_1(\xi, Z, \text{Fo})\xi^2 - \xi^3 Z\exp(-\xi Z)\Phi_2(\xi, 0, \text{Fo})\right]J_0(\xi\rho)\,d\xi,$$

$$\sigma^t_{rz} = C\int_0^\infty \xi^2\varphi(\xi)\left[\Phi_2(\xi, Z, \text{Fo}) - (1 - \xi Z)\exp(-\xi Z)\Phi_2(\xi, 0, \text{Fo})\right]J_1(\xi\rho)\,d\xi.$$

The total stress field caused by turning friction at an arbitrary point $X(r, z)$ of the body is expressed as the sum

$$\sigma_{ij} = p_0\left[\sigma^{e*}_{ij}(X) + w_t\sigma^{t*}_{ij}(X, \text{Fo})\right] \quad (i, j = r, \theta, z). \tag{20}$$

Here $\sigma^{e*}_{ij} = \sigma^e_{ij}/p_0$, $\sigma^{t*}_{ij} = \sigma^t_{ij}/C$ are the dimensionless stresses in Hertz's isothermal problem [2] and thermal stresses (19), respectively. In the case of friction contact of turning, the level of the stressed state of the heated body will be determined in terms of the second invariant of the stress deviator J_2 (the Huber–Mieses stress [7]):

$$J_2 = \left\{\frac{1}{6}\left[(\sigma_{rr} - \sigma_{\theta\theta})^2 + (\sigma_{\theta\theta} - \sigma_{zz})^2 + (\sigma_{zz} - \sigma_{rr})^2\right] + (\sigma_{rz})^2\right\}^{1/2},$$

where the stress components σ_{ij} are given by formulas (20).

Figure 2 shows the calculated levels of the dimensionless stress levels $J_2^* = J_2/J_0$ plotted at $\nu = 0.3$, Fo = 10. It is seen, for example, that for $w_t = 3$ the maximum stress level J_2^* is lower than the stress level at $w_t = 0$ (the heat release is zero), while at $w_t = 8$ the stress concentration in the body exceeds the stress concentration in the isothermal problem. This is confirmed by the data presented in Fig. 3, where the maximum $J^*(J_{2\max}^*)$ is plotted versus the parameter w_t. It is clearly seen that there exists a local minimum of $J_{2\max}^*$ attained at a certain (critical) value of the parameter w_t. As follows from (18), its expression contains both an invariable side (mechanical and thermophysical properties of the material) and terms caused by the contact (γ, f, ω). The present numerical analysis shows that for a preset friction pair conditions at the contact can be chosen such that the overall level of the stressed state of the body due to friction heating is decreased in comparison with the case of contact ignoring heat release. The results of numerical studies presented in Figs. 2 and 3 also show that at a low level of thermal stresses a maximum Huber–Mieses stress develops inside the body and with increase in the thermal stress (increase in the parameter w_t) it emerges at the surface of the body.

The present data can be a basis for study of the conditions of the development of plastic flow in local regions of contacting bodies.

NOTATION

f, friction coefficient; k, K, thermal diffusivity and thermal conductivity; $\Delta = \partial/\partial r^2 + (1/r \cdot \partial/\partial r) + \partial^2/\partial z^2$, Laplace operator; α, linear thermal expansion coefficient; ν, Poisson coefficient; μ, shear modulus; T, temperature; $J_\nu(\cdot)$, first order Bessel function of ν; erf $(\cdot)$, probability integral.

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